SLIET LONGOWAL

2.2.8 - MATHEMATICS (Ph.D. - MATH)

Course Work Subject Codes and Subject Names

SUBCODE	SUBJECT NAME
AM-10001	UNIVALENT FUNCTIONS
AM-10002	MATRIX ANALYSIS
AM-10003	NUMERICAL ANALYSIS & C++ PROGRAMMING
AM-10004	ADVANCED NUMERICAL ANALYSIS
AM-10005	C PROGRAMMING
AM-10006	WAVELET ANALYSIS
AM-10007	STATISTICS AND FINANCE MATHEMATICS
AM-10008	DIFFERENTIAL GEOMETRY
AM-10009	COSMOLOGY
AM-10010	ADVANCE COSMOLOGY
AM-10011	COSMIC TOPOLOGY
AM-10012	MATHEMATICAL METHODS
AM-10013	STATISTICAL TECHNIQUES

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10001 UNIVALENT FUNCTIONS

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UNIT- I

Normal families of analytic functions, Montel's theorem, Hurwitz's theorem, Riemann Mapping theorem. Analytic continuation, analytic continuation by power series, Natural boundary. Schwarz reflection principle, Analytic continuation along a path. Monodromy theorem.

UNIT- II

Univalent function. Area theorem, distortion and growth theorem for the class of normalized univalent functions, Koebe-1/4 theorem, Starlike functions, Convex functions, Spirallike functions, Typically real functions, Growth of integral means, Asymptotic Bieberbach conjecture, Schwarz's Lemma, Subordination principle, Univalent subordinate functions.

UNIT- III

Bounded univalent functions, Sections of univalent functions, convolution of convex functions, coefficient multipliers, Criteria for univalence, Functions of bounded boundary rotation.

UNIT- IV

Harmonic functions, basic properties, Harmonic functions on a disc, Subharmonic and superharmonic functions, The Dirichlet problem, Green's function.

RECOMMENDED BOOKS

1. Zeev Nihari, Conformal Mapping.
2. E. T. Copson, An Introduction to Theory of Functions of a Complex Variable.
3. J. B. Conway, Functions Of One Complex Variable.
4. T. W. Gamelin, Complex Analysis.
5. P. L. Duren, Univalent Functions.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10002 MATRIX ANALYSIS

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Review of Linear Algebra

Vector spaces and Inner product spaces; Linear Operator and Matrices consisting of unitarily equivalent triangular form of a matrix, diagonal form of normal matrix, spectral Theorem, Hermitian matrix, positive matrix and its decompositions, Direct sums, block matrices, Tensor product and Hadamard product.

Majorisation and doubly stochastic matrices

Notion and related basic results of majorisation of vectors; doubly stochastic, substochastic matrices and related results to the majorisation; Birkhoff's theorem; convex and monotone functions and related results to the majorisation of vectors and doubly stochastic, substochastic matrices; Symmetric gauge functions.

Variational Principles for Eigenvalues

The Minmax Principle for Eigenvalues, Poincare's Inequality, Cauchy's Interlacing Theorem, Weyl's Inequalities; Wielandt's Minmax Principle; Lidskii's Theorems; Eigenvalues of real parts and Singular values, Fan Hoffman Theorem.

Symmetric Norms

Norms on ", Unitarily Invariant Norms on Operators on ", Symmetric Gauge Function, Fan Dominance Theorem, Cauchy Schwarz Inequality for Unitarily invariant Norms, Quadratic Symmetric Gauge Functions, Weakly Unitarily Invariant Norms,

Operator Monotone and Operator Convex Functions

Definitions of Matrix Monotone and Matrix Convex Functions, Some equivalent conditions, First and Second divided differences, Smoothness Properties, Relation between Derivative and Matrix Functions, Loewner's Theorems.

Books Recommended:

1.   R. Bhatia, *Matrix Analysis,* Springer-Verlag .
2.   R.A. Horn and C.R. Johanson, *Matrix Analysis,* Cambridge University Press.
3.   R.A. Horn and C.R. Johanson, *Topics in Matrix Analysis,* Cambridge University Press.

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AM-10003 NUMERICAL ANALYSIS & C^{++} PROGRAMMING

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Unit-I

Non linear single equation : Secant method, Successive approximation method, Newton-Raphson method, Muller method, System of non linear equations : Newton-Raphson method, Successive approximation method, Finite differences, Newton's forward, backward and divided difference interpolation formulae, Lagrange's formula, Gauss's formula, Bessel's formula, Hermite's interpolation, Cubic spline interpolation.

Unit-II

Numerical differentiation using, Newton's forward difference, backward difference and cubic spline method, Introduction to quadrature, Newton Cotes quadrature formulae, Simpson's formulae, Romberg integration, Modified Euler's method, Runge-Kutta methods, Predictor Corrector techniques, Stability Analysis, Finite difference technique.

Unit-III

Introduction to C^{++}, Basic structure of C^{++} programs, Executing a C^{++} program, Constants, Variables, Declarations of variables, Different types of operators like: arithmetic, relational, logical, assignment, conditional, increment and decrement, Arithmetic expressions, Evaluation of expressions, Precedence of arithmetic operators, Mathematical functions, Control statements: if-else statement, nested if....else statements and else if ladder, while statement, do statement, for statement, jumps in loops, switch statement, break statement, continue statement, conditional statement and go-to statement.

Unit-IV

Defining an array, One-dimensional arrays, Two-dimensional arrays, Initializing two dimensional arrays, Multidimensional arrays, Strings, Defining a functions, A multi-function program, Return values and their types, Calling a function, No arguments and no return values, Arguments but no return values, Arguments with return values, Handling of non-integer functions, Understanding pointers, Accessing the address of a variable, Declaring and initializing pointers, Accessing a variable through its pointer, Pointer expressions.

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Recommended Books

1. Introductory Method of Numerical Analysis, S.S. Sastry, Prentice Hall India.
2. Numerical Methods for Engineers, S.K. Gupta, New Age Int (P) Ltd., Publishers.
3. Applied Numerical Analysis, C.F. Gerald & P.O. Wheatley, Pearson Education.
4. Numerical Methods for Scientific and Engineering Computation, M.K. Jain, S.R.K. Iyengar and R.K. Jain, New Age International
5. Object-Oriented Programming with C^{++}, E. Balagurusamy, Tata McGraw-Hill.
6. SCHAUM'S OUTLINES of Theory and Problems of Programming with C^{++}, John R. Hubbard, McGraw-Hill

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10004   ADVANCED NUMERICAL ANALYSIS

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Unit-I

Fixed point method, Multi variable successive substitution method, Single variable Newton-Raphson Technique, Multi variable Newton-Raphson Technique, Lin Bairstow technique.

Unit-II

Numerical differentiation using Newton's forward difference, backward difference and cubic spline method. Introduction to quadrature, Newton Cotes quadrature formulae, Trapezoidal, Simpson's, Boole's and Weddle's rules, Romberg integration, Gauss Legendre integration.

Unit-III

Taylor series method, Euler method, Modified Euler's method, Runge-Kutta methods, Predictor Corrector techniques, Adam-Bashforth method for multi variable ODE IVPs, Stiffness of ODEs, Gear's technique for Stiff ODEs.

Unit-IV

Finite difference technique, Orthogonal collocation, Orthogonal collocation on finite elements, Galerkin finite element technique for the solution of ODEs and PDEs.

Recommended Books

1.   Elements of Numerical Analysis, R.S. Gupta, Macmillan India Ltd.
2.   Numerical Methods for Engineers, S.K. Gupta, New Age Int (P) Ltd., Publishers.
3.   Numerical Methods for Mathematics, Science & Engg, J.H. Mathew, Prentice Hall India.
4.   Applied Numerical Analysis, C.F. Gerald & P.O. Wheatley, Pearson Education.
5.   Numerical Methods for Scientific and Engineering Computation
6.   M.K. Jain, S.R.K. Iyengar and R.K. Jain, New Age International.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10005 C PROGRAMMING

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Unit-I

Introduction to C, basic structure of C programs, executing a C program, data types, constants, variables, declarations of variables, assigning values to variables, defining symbolic constants, different types of operators like-arithmetic, relational, logical, assignment, conditional, increment and decrement, bitwise, special, arithmetic expressions, evaluation of expressions, precedence of arithmetic operators, mathematical functions.

Unit-II

Single character input, getchar function, Single character output, putchar function, Entering input data, scanf function, Writing output data, printf function, formatted input, formatted output, decision making with if statement, if-else statement, nested if....else statements, else if ladder, while statement, do statement, for statement, jumps in loops, switch statement, break statement, continue statement, conditional statement, goto statement.

Unit-III

Defining an array, one-dimensional arrays, two-dimensional arrays, initializing two dimensional arrays, multidimensional arrays, declaring and initializing string variables, reading strings from terminal, writing strings to screen.

Unit-IV

Defining a functions, A multi-function program, return values and their types, calling a function, no arguments and no return values, arguments but no return values, arguments with return values, handling of non-integer functions, understanding pointers, accessing the address of a variable, declaring and initializing pointers.

Recommended Books
1.   Programming with C, B.S. Gottfried, Tata McGraw Hill.
2.   Programming in ANSI C, E. Balagurusamy, Tata McGraw Hill.
3.   C tools for Scientists and Engineers, L. Barker, McGraw Hill.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10006 WAVELET ANALYSIS

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UNIT-I

Vector spaces, Inner products, Orthonomal bases, Riesz systems and frames. $L^2(R)$ and approximate identities. Fourier transform, Basic Properties. Fourier inversion. Discrete Fourier transform.

UNIT-II

Continuous wavelets. Continuous wavelet transform (CWT) and properties. Inverse CWT. Discrete wavelet transform. Haar scaling function and wavelet function , Scaling equations.

UNIT-III

Evaluations of Φ,. Multiresolution analysis (MRA). Construction of a general orthonormal MRA. Wavelet basis for MRA. Daubechies wavelets. Support of a wavelet system.

UNIT-IV

Condition number of matrix. Application of wavelets(certain case): Wavelet -Galerkin mathods for differential equations.

Reference Books

1.   R.M. Rao and A.S Bopardikar, Wavelet Transforms: Introduction to Theory and Applications, Pearson Education.
2.   Brani Vidakovic, Statistical Modeling by Wavelets, John Wiley & Sons.
3.   A.I. Zayed, Function and Generalized Function Transforms, CRS Press.
4.   M.W. Frazier, An Introduction to Wavelets Through Linear Algebra, Springer.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10007 STATISTICS AND FINANCE MATHEMATICS

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Unit -1

Probabilities and Events, Conditional probability, Random Variable and Expected values, Normal Random Variable, Properties of normal random variables. The central limit theorem, Geometric Brownian motion, Geometric Brownian motion as a limit of simple models, Brownian motion.

Unit-2

Introduction and definition of Index Numbers, Uses of Index Numbers, Problems in the Construction of Index Numbers, Different Method of constructing Index Numbers, Tests of Adequacy of Index Numbers formulae, The Chain Index Numbers, Index Numbers of Industrial Production, Introduction and definition of time series analysis, Utility of time series, Components of time series analysis, measurement of trend, Measuring theory by logarithms. Measurement of seasonal variations, Measurement of cyclical variations, Measurement of irregular variations.

Unit-3

Interest rates, Present value analysis, Rate of return, Continuously varying interest rates, An example in options pricing, Other examples of Pricing via Arbitrage, The Arbitrage Theorem. The multi period binomial model, Proof of the Arbitrage Theorem, The Black- Scholes Formula, Properties of the Black- Scholes option cost, The delta hedging Arbitrage strategy, Some derivations.

Unit-4

Call option on dividing-paying securities, adding jumps to Geometric Brownian Motion, limitation of Arbitrage pricing, valuing investment by expected utility, Value at risk and conditional value at risk, The Capital Assets Pricing Model, Deterministic Optimization Models. Probabilistic optimization problems.

Reference Books
1. Sheldon M. Ross, An Elementary Introduction to Mathematical Finance, Cambridge University Press.
2. James D. Hamilton, Time Series Analysis, Princeton University Press.
3. Goon Gupta and Das Gupta, Fundamentals of statistics.
4. Raiford Theodore, Mathematics of Finance.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10008 DIFFERENTIAL GEOMETRY

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UNIT-I

Tensor and their transformation laws, Tensor algebra, Contraction, Quotient law, Reciprocal tensors, Kronecker delta, Symmetric and skew- symmetric tensors, Metric tensor, Riemannian space, Christoffel symbols and their transformation laws, Covariant differentiation of a tensor, Riemannian curvature tensor and its properties, Bianchi identities, Ricci-tensor.

UNIT-II

Theory of space curves. Serret-Frenet formulas for curves in space. Parametric representation of curves, Helix, Curvilinear coordinates in E_3. Tangent and first curvature vector. Intrinsic equations & differentiation, Parallel vector fields.

UNIT-III

Parametric representation of a surface, Tangent and Normal vector field on a surface, The first and second fundamental tensor. The third fundamental form, Gaussian curvature , Isometry of surfaces, Developable surfaces, Weingarten formula, Equation of Gauss and Codazzi , Principal curvature, Normal curvature, Meusnier's theorem, Rodrigue's formula, Asymptotic lines.

UNIT-IV

Definition. Differential equation of geodesics. Nature of Geodesics. Canonical equations. Normal property. Geodesic polar coordinate, curvature and torsion.

RECOMMENDED BOOKS

1.   I.S. Sokolnikoff, Tensor Calculus and Application to Geometry and Mechanics, (chapter-II and III).
2.   T.T.Wilmore, An Introduction to Differential Geometry, (chapter – I,II,III,V and VI).
3.   Bary Spain, Differential Geometry.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10009 COSMOLOGY

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UNIT-I

Preliminaries cosmology, Homogeneous and isotropic universe. The cosmological principle, General relativistic cosmological models. The problem of cosmology - emphasizing global (space-time structure of universe) and local properties.

UNIT-II

Observational Cosmology: Galaxies and clusters; mass density; Friedman cosmological models,cosmology with non zero lambada. Hubble's law and its implications; cosmic background, radiation - its discovery and implications. Cosmological implications of General Relativity.

UNIT-III

The Einstein field equations; the Schwarzschild and Robertson-Walker metrics as examples of local and global physics. The Cosmological Principle Expansion of Universe, Robertson-Walker metric (RW metric), time dilation, redshift holiday , Friedmann equation, equation of state Cosmic dynamics, evolution of universe. Theory behind variable cosmology.

UNIT-IV

Model Cosmologies: The 3 Friedman universes and their connection with the Einstein-de Sitter universe; importance of deceleration parameter in deciding between different possible models, the ê-factor a question of "missing matter". Gravitational collapse of a homogeneous dust ball. Simple idea of black hole physics.

RECOMMENDED BOOKS

1. J.V. Narlikar, General Relativity & Cosmology.
2. B.F. Schultz, A First Course in General Relativity.
3. Barbara Ryden, Introduction to Cosmology.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10010 ADVANCE COSMOLOGY

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UNIT-I

Preliminaries of Differential Geometry, Curvilinear coordinates, Christoffel symbols, principal curvature, Line of curvature, the Duplin indicatrix, Developable surfaces Geodesics and its properties, its differential equations, Existence theorems, Gaussian curvature, Conformal mapping, Tensor notations, Ricci tensor and its properties

UNIT-II

Preliminaries cosmology, Homogeneous and isotropic universe. The cosmological principle, General relativistic cosmological models. Observational Cosmology: Galaxies and clusters; mass density; Friedman cosmological models, cosmology with non zero lambada. Hubble's law and its implications; cosmic background, radiation - its discovery and implications. Cosmological implications of General Relativity.

UNIT-III

Concept of Dark energy and Dark matter in the universe.The Einstein field equations; the Schwarzschild and Robertson-Walker metrics as examples of local and global physics. The Cosmological Principle Expansion of Universe, Robertson-Walker metric (RW metric), time dilation, Bianchi type –IX, model.

UNIT-IV

Invariant Lyapunov exponents and Chaos in cosmology, Asymptotics and singularities in cosmological models with positive cosmological constant, Volume and density parameters of the universe

RECOMMENDED BOOKS
1. J.V. Narlikar, General Relativity & Cosmology.
2. B.F. Schultz, A First Course in General Relativity.
3. Barbara Ryden, Introduction to Cosmology.
4. D. Somasundaram, Differential Geometry

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10011 COSMIC TOPOLOGY

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UNIT-I

Preliminaries cosmology, Homogeneous and isotropic universe. The cosmological principle, General relativistic cosmological models. The problem of cosmology - emphasizing global (space-time structure of universe) and local properties.

UNIT-II

Observational Cosmology: Galaxies and clusters; mass density; Friedman cosmological models, cosmology with non zero lambada. Hubble's law and its implications; cosmic background, radiation - its discovery and implications. Cosmological implications of General Relativity.

UNIT-III

The Einstein field equations; the Schwarzschild and Robertson-Walker metrics as examples of local and global physics. The Cosmological Principle Expansion of Universe, Robertson-Walker metric (RW metric), time dilation, redshift holiday , Friedmann equation, equation of state Cosmic dynamics, evolution of universe. Theory behind variable cosmology.

UNIT-IV

The complete metric spaces, the compactness in metric spaces, point wise and compact convergence. Open maps, closed maps and Homomorphism, Product topology, Quotient topology, metric topology, complete metric space. Connected spaces

RECOMMENDED BOOKS

1. J.V. Narlikar, General Relativity & Cosmology.
2. B.F. Schultz, A First Course in General Relativity.
3. Barbara Ryden, Introduction to Cosmology.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10012 MATHEMATICAL METHODS

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Unit-I

Complex Analysis:

Analytical functions, confomal mapping, singularities and residues, Cauchy theorem, Cauchy integral formula, Poission formula, Taylor's and Laurent series, Cauchy residue theorem and evaluation of complex integral, Analytical continuation Rouche's theorem and Morera's theorem.

Unit –II

Thermoelasticity and Waves:

Analysis of strain and stresses, types of strains and stresses, Elastic moduli generalized Hooke's law and its reduction to isotropic media and connection between them, Strain energy density function, Equation of equilibrium and motion, elastic waves in solids and their nomenclature, Thermoelasticity, Duhmal – Neumann relations, Thermoelastic waves and stresses, Viscoelasticity, viscoelastic waves and plane stresses and their problems, Circular, cylindrical and spherical solid bodies.

Unit-III

Numerical Method Techniques:

Least square curve fit and spline interpolation,Numerical methods in ordinary differential equations: Runge – Kutta methods, Pridictor – Corrector methods, Simaltanous and higher order equations, Boundary value problems.

Finite difference method: Ordinary differential equations, shooting method, cubic spline method, partial differential equation: five point standard formula, diagonal five point formula for elliptic equations, Schmidt method, Crank – Nicolson methods, Du – Fort and Frankal method for parabolic equations and explicit schemes for solving hyperbolic equations.Finite Element Method: Rayleigh–Ritz method, Galerkin method and its applications

Unit-IV

Numerical Programming:

Numerical programming and software MATLAB, MATHCAD, MATHEMATICA, FORTRAN and C++ for solving Boundary Value Problem / Initial Value Problems.

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Referred Books:

1. Mathematical theory of Elasticity: by I. S. Sokololnikoff, Tata Mc Graw-Hill, New Delhi.
2. Dynamic Theory of Thermoelasticity, W. C. Nowacki, (2nd ed.). Warszawa: PWN-PolishScientific Publishers.
3. Numerical Methods of Scientists and Engineers, By J. N. Sharma Narosha Publications.
4. Getting Started with MATLAB By Rudra Pratap Oxford University Press. 5. A first course in complex analysis with applications, 2e by Dennis G. Zill and Patrick D.Shanahan.

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2.2.8 - MATHEMATICS (Ph.D. - MATH)

AM-10013 STATISTICAL TECHNIQUES

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UNIT - I

Karl-Pearson coefficient of correlation. Partial and multiple correlation (three variables case only). Regression analysis up to three variables. Review of probability. Discrete and continuous random variables. Mathematical expectations of a random variable. Probability mass function. Probability density function. Probability distribution function. Functions of a random variable. Discrete and continuous univariate distributions - Binomial, Poisson, Normal, exponential.

UNIT - II

Moments, Moments generating functions, Characteristic function, Joint distribution function, Marginal and conditional distributions, Joint density function, Marginal and conditional density functions.

UNIT - III

Sampling distribution and testing: Hypotheses, types of errors, critical and acceptance regions, level of significance. Sampling distribution, Standard error. Chi-square distribution, Student's t distribution and F distribution. Fundamental notions, Tests based on normal, t, Chi-square and F distributions. Analysis of variance: One-way and two-way classifications. Non parametric tests: Sign test, one sample Wilcoxon signed rank test, Mann-Whitney test, Kolmogorov-Smirnov test, Kruskal Wallis test, Friedman test.

UNIT - IV

Simple random sampling, systematic sampling, stratified sampling, cluster sampling, sampling with probability proportional to size. Multivariate techniques: Discriminant analysis, Factor analysis, Principal component analysis, Cluster analysis, Response Surface Methodology.

RECOMMENDED BOOKS:
1. T.W. Anderson, An Introduction to Multivariate Statistical Analysis, John Wiley (2003).
2. S.P. Gupta, Statistical Methods, Sultan Chand & Sons (2017).
3. S.C. Gupta & V.K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons (2014).
4. R.A. Johnson & D.W. Wichern, Applied Multivariate Statistical Analysis, PHI (2012).
5. D.F. Morrison, Multivariate Statistical Analysis, McGraw Hill (1990).